

Roll No.

S-2133

**M. A./M. Sc. (Second Semester)
EXAMINATION, 2018**

MATHEMATICS

Paper Second

(Fluid Dynamics)

(MATH-C-008)

Time : Two Hours] [Maximum Marks : 60

Note : Attempt any four questions. All questions carry equal marks.

1. (a) Define any five of the following :
 - (i) Compressible liquid
 - (ii) Flux across any surface
 - (iii) Perfect liquid
 - (iv) Fluid pressure
 - (v) Stream line
 - (vi) Ideal and Real fluid
- (b) Defining the local and individual time rate of changes find the relation between them.
2. (a) State and prove Bernoulli's equation.

- (b) Show that the variable ellipsoid :

$$\frac{x^2}{a^2 k^2 t^4} + k^2 \left[\left(\frac{y}{b} \right)^2 + \left(\frac{z}{c} \right)^2 \right] = 1$$

is a possible form for the boundary surface of a liquid of time t .

3. (a) Defining the complex potential and image, state and prove Milne-Thomson circle theorem.
- (b) An infinite mass of fluid is acted on by a force $\left(\frac{\mu}{r^{3/2}} \right)$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere $r = c$ in it, show that the cavity will be filled up after an interval of time $\left(\frac{2}{5} \mu \right)^{1/2} c^{5/4}$.
4. (a) State and prove Kelvin's minimum energy theorem.
- (b) Show that theorem under certain conditions, the motion of a frictionless fluid if once irrotational, will always be so, is true also when each particle is acted on by a resistance varying as the velocity.
5. (a) Defining the connectivity and simply connected region show that the necessary and sufficient condition for irrotational motion is that there exists a velocity potential ϕ such that $q = -\nabla\phi$. q being velocity vector.

- (b) What arrangement of sources and sinks will give rise to the function $W = \log \left(z \frac{-a^2}{z} \right)$. Draw a rough sketch of the stream lines prove that two of the stream lines subdivide into the circle $r = a$ and the axis of y . <http://www.hnbguonline.com>

6. (a) A circular cylinder is moving in a liquid at rest at infinity, calculate the forces on the cylinder owing to the pressure of the liquid.

- (b) Use the method of images to prove that if there be a source 'm' at a point z_0 in a fluid bounded by the lines $\theta = 0$ and $\theta = \frac{\pi}{3}$, the solution is :

$$\phi + i\psi = -m \log \left\{ (z^3 - z_0^3)(z^3 - z_0'^3) \right\} \text{ where } z_0 = x_0 + iy_0 \text{ and } z_0' = x_0 - iy_0.$$

7. (a) When the external forces are conservative and derivable from a single valued potential function of pressure only, then prove that the circulation in any closed circuit moving with the fluid is constant for all time.

- (b) Between the fixed boundaries $\theta = \frac{\pi}{6}$ and

$\theta = -\frac{\pi}{6}$, there is a two dimensional liquid motion due to a source at the point $r = c, \theta = \alpha$, and a sink at the origin, absorbing water at the

same rate as the source produces it. Find the stream function and show that one of the stream lines is a part of the curve $\gamma^3 \sin 3\alpha = c^3 \sin 3\theta$.

8. (a) A circular cylinder is fixed across a stream of velocity U with circulation k round the cylinder. Show that the maximum velocity in the liquid is

$$2U + \frac{k}{2\pi a}, \text{ where } a \text{ is the radius of the cylinder.}$$

- (b) The space between two infinitely long coaxial cylinders of radii 'a' and 'b' respectively is filled with homogeneous liquid of density P and is suddenly moved with velocity U perpendicular to the axis, the outer one being kept fixed. Show that the resultant impulsive pressure on a length 'l' of the cylinder is :

$$\pi \rho a^2 l \frac{b^2 + a^2}{b^2 - a^2} U$$

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