Roll No. ....

S–2132

M.A./M. Sc. (Second Semester) EXAMINATION, 2018

MATHEMATICS

Paper First

(Abstract Algebra—II)

(Math—C—007)

Time : Two Hours] [*Maximum Marks : 60*

Note : Attempt any *four* questions. All questions carry equal marks.

1. (a) Define the following terms with example :

   (i) Simple field extension

   (ii) Algebraic element

   (b) Let K be an extension of a field F and $p(x)$ a minimal polynomial satisfied by $a \in K$. Then prove that $p(x)$ is irreducible over F.

2. (a) Let K be an extension of a field F. Then the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

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   (b) If $a, b \in K$ are algebraic over a field F of degrees m and n respectively, and if m and n are relatively prime, prove that $F(a, b)$ is of degree $m.n$ over F.

3. Let K be an extension of a field F and $a \in K$ algebraic over F satisfied by the minimal polynomial $p(x)$ over F. Then prove that $F(a)$ is isomorphic to $F[x]/V$ where V is the ideal generated by $p(x)$.

4. (a) Let $f(x) = x^4 + x^2 + 1$ is a polynomial over the field of rational number Q. Then find the splitting field of $f(x)$ over Q and also find its degree.

   (b) If E is an extension of F and if $f(x) \in F[x]$ and ϕ is an automorphism of E leaving every element of F fixed, prove that ϕ must take a root of $f(x)$ lying in E into a root of $f(x)$ in E.

5. If K is a field and if $\sigma_1, \sigma_2, ..., \sigma_n$ are distinct automorphism of K, then prove that it is impossible to find elements $a_1, a_2, ... a_n$ not all zero in K such that :

$$a_1\sigma_1(u) + a_2\sigma_2(u) + ... + a_n\sigma_n(u) = 0 \; \forall u \in K$$

6. (a) Define the following terms with example :

   (i) Separable extension

   (ii) Fixed field

   (b) Prove that any finite extension of a field F of characteristic zero is a simple extension.

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7. **Prove that the Galois group of a polynomial over of a** field F is isomorphic to a group of permutations of its roots.

8. (a) If the real number α satisfies an irreducible polynomial over the field of rational numbers of degree k, and if k is not a power of 2, then prove that α is not constructible

   (b) Prove that the general polynomial of degree $n \geq 5$ is not solvable by radicals.