Roll No. ....

233312

S–251(B)

B. A./B. Sc. (Fourth/Sixth Semester)

EXAMINATION, 2021-22

(Skill Enhancement Course)

MATHEMATICS

(Vector Calculus)

(SOS/Maths/SEC–002)

Time : Two Hours] [*Maximum Marks : 70*

Note : (i) Attempt any *five* questions (out of seven questions) from Section A and any *three* questions (out of six questions) from Section B.

(ii) Answer each question of Section A within 50 words.

(iii) Limit your answers within the given answer book. Additional answer book (B-Answer book) should not be provided or used.

P. T. O.

Section—A

Note : Attempt any *five* questions. Each question carries 5 marks.

1. If a, b, c are position vectors of A, B and C, then prove that :

$$a \times b + b \times c + c \times a$$

is a vector perpendicular to the plane of ABC.

2. Find the value of p so that the vectors :

$$2i - j + k$$

$$i + 2j - 3k$$

and $$3i + pj + 5k$$

are coplanar.

3. If vectors a', b' and c' are reciprocal systems to the vectors a, b and c then prove that :

$$[a'\,b'\,c'] = \frac{1}{[a\,b\,c]}$$

4. The necessary and sufficient condition that vector $a(t)$ is a vector of constant magnitude is :

$$a.\frac{da}{dt} = 0$$

5. Find grad f where $f = x^3 + y^3 + 3xyz$.

6. If $f = xyi + yzj + zxk$, then prove that :

$$\nabla^2 f = 0$$

7. Give the statement of Green's theorem.

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Section—B

Note : Attempt any *three* questions. Each question carries 15 marks.

8. (i) If the four vectors a, b, c and d are coplanar, then prove that :

$$(a \times b) \times (c \times d) = 0$$

(ii) Find r from the equation :

$$\frac{d^2 r}{dt^2} = at + b,$$

given that both r and $\frac{dr}{dt}$ vanish when $t = 0$.

9. A particle moves along the curve $x = t^3 + 1$, $y = t^2$ and $z = 2t + 5$ where t is the time. Find the component of its velocity and acceleration at $t = 1$ in the direction $i + j + 3k$.

10. (i) If a and b are vectors then prove that :

$$\text{grad } (a.\, b) = a \times \text{curl } b + b \times \text{curl } a + (a.\, \nabla)\, b + (b.\, \nabla)\, a$$

(ii) $\text{div } (a \times b) = b.\ \text{curl } a - a.\ \text{curl } b$, a and b are vectors.

11. Evaluate :

$$\int_C F.dr$$

where $F = xyi + yzj + zxk$ and curve C is $r = it + jt^2 + kt^3$, t varying from -1 to $+1$.

12. Using Gauss's divergence theorem, prove that :

$$\int_S (axi + byj + czk).n\, dS = \frac{4}{3}\pi(a + b + c)$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

13. Verify Stokes' theorem for the function $F = zi + xj + yk$ where curve is the unit circle in the xy-plane bounding the hemisphere $z = \sqrt{1 - x^2 - y^2}$.

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